An implicit method for transient gas flows in pipe networks

Tatsuhiko Kiuchi
Toyo Engineering Corporation, Chiba, Japan

This paper describes a fully implicit finite-difference method for calculating the unsteady gas flow in pipeline networks. The algorithm for solving the finite-difference equations of a pipe is based on the Newton–Raphson method. The Von Neumann stability analysis on the finite-difference equations of a pipe shows that the equations are unconditionally stable. An iterative convergence method is applied to the calculation of node pressure at junctions in networks. The parameter study on the convergence shows that the stability depends on the convergence tolerance. Calculation results of a few sample cases are compared with those of the method of characteristics and the two-step Lax–Wendroff method. An excellent agreement between the methods is obtained when a small time step is used. Computation time can be greatly reduced by using the implicit method.

Keywords: pipeline; fluid transients; implicit method; compressible flow

1. Introduction

The numerical calculation of unsteady pipe flow plays an important role in the design of petrochemical plants, hydraulic power machines, long distance pieplines, etc., and has become popular in these industrial areas due to the development of high-speed computers.

Reviewing calculation methods, we find traditional explicit methods such as the method of characteristics (MOC) (Wylie and Streeter 1978), the Lax–Wendroff method (Bender 1979), and the two-step Lax–Wendroff method (Poloni et al. 1987). However, Fincham and Goldwater (1979) pointed out that these methods have an obvious disadvantage in calculations concerning large pipeline networks, because the time-steps are restricted by the Courant condition.

The main purpose of this paper is to demonstrate an appropriate technique for the predictive simulation of large, complicated city gas networks that require fast computation to predict long-term behavior over a few days. For this purpose, several implicit methods have been developed. Guy (1967) proposed a partially implicit algorithm based on the Crank–Nicolson method, in which the continuity and momentum equations are applied, node to node, alternatively; Guy solved the equations by the calculation of a tridiagonal matrix. Rachford et al. (1975) developed an implicit technique using the Galerkin method. Schmidt (1977) used the Crank–Nicolson and the backward Euler methods alternately to avoid spurious oscillations (see Chua 1982).

Osiadacz (1984) applied a fully implicit method to the linearized equations, which are obtained by neglecting inertia terms. Some of these methods are already in use in the gas industry. However, there is very limited information available concerning instability of solutions and comparison with other methods.

This paper aims to present another fast, stable, and practically accurate implicit method for the calculation of the transient gas flow in large pipeline networks. The method is based on a fully implicit algorithm that includes the calculation of the inertia terms. The von Neumann stability anlaysis (see Roache, 1985) shows that this method is unconditionally stable, while the Crank–Nicolson method is conditionally stable. Comparison with explicit methods indicates that the proposed method gives good solutions in both rapid and slow transient phenomena, although rapid transients are filtered if large time-steps are used. Comparison with Guy's method indicates that the proposed method has an advantage in terms of computation time and stability.

2. Calculation method

2.1. Basic equations

To describe the fluid motion in a gas pipeline, the following assumptions are made: 1) 1-D isothermal compressible flow, 2) steady-state friction, and 3) negligible expansion of pipe wall due to pressure changes. These assumptions have been discussed and are commonly used by many authors (for example, Chua 1982). The continuity and momentum equations written in the convection form are given by

$$p_t + c^2 m_x = 0 \tag{1a}$$

$$m_t + (m^2 c^2/p)_x + p_x + (fc^2/2D)m|m|/p + pS = 0 \tag{1b}$$

where the friction factor f is given as a function of Reynolds number, and $S(=(g/c^2)\sin\phi)$ designates the gravity term, which can be negligible in the calculation of gas flow. Under the assumption of isothermal flow, the speed of sound c is $\sqrt{zRT}$. Compressibility factor z is calculated in the present method by the simple equation for natural gas (see Reet and Skogman 1987), which is a function of pressure and temperature in reasonable ranges.

The first and second terms in Equation 1b are inertia terms. The second term is neglected in the present paper, assuming a small flow velocity compared to the sound speed. This assumption is reasonable because the ratio of the values among the pressure term and the first and second inertia terms is approximately 1:0.1:0.01 for most cases of operations in actual gas pipelines (see Guy, 1967; Osiadacz, 1984).

Address reprint request to Dr. Kiuchi at Toyo Engineering Corporation, 8-1, Akanehama, 2-chome, Narashino-shi, Chiba, 275, Japan.

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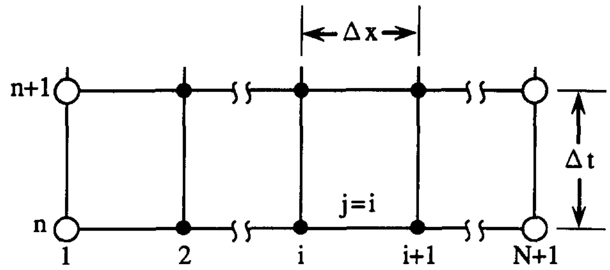


Figure 1 Mesh of the calculation

2.2. Differential method

Figure 1 shows a mesh used in the calculation. A pipe is divided into N sections. Section j is defined as the section between node i and $i + 1$. n means time level. Equations 1a and 1b are applied to section j by using the centered-difference form in space and a fully implicit algorithm in time. The difference forms for Equations 1a and 1b become

$$ {}_jH = (p_{i+1}^{n+1} + p_i^{n+1} - p_{i+1}^{n} - p_i^{n})/(2\Delta t) + (m_{i+1}^{n+1} - m_i^{n+1})c^2/\Delta x \quad (3a)$$

$$ {}_jG = (m_{i+1}^{n+1} + m_i^{n+1} - m_{i+1}^{n} - m_i^{n})/(2\Delta t) + (p_{i+1}^{n+1} - p_i^{n+1})/\Delta x + (fc^2/4D) \times |m_{i+1}^{n+1} + m_i^{n+1}| \times (m_{i+1}^{n+1} + m_i^{n+1})/ (p_{i+1}^{n+1} + p_i^{n+1}) + (p_{i+1}^{n+1} + p_i^{n+1})S/2 \quad (3b)$$

Applying Equations 3a and 3b to each section, $2N$ equations are derived for a pipe. The number of unknown values at time level $n + 1$, which consists of pressure and mass flux at each node, is $2(N + 1)$. Since the pressure or mass flux at both ends of a pipe is given by boundary conditions or by the iterative method described in section 2.4, the number of unknown values is reduced to $2N$. Thus, the Newton–Raphson method can be applied to solve these simultaneous equations of a pipe.

2.3. Stability anlaysis

The Crank–Nicolson method is not always stable, as shown in section 3. However, the Von Neumann stability analysis shows that the differential equations (Equations 3a and 3b) of this method are unconditionally stable. Dimensionless expressions of these equations using $t^* = t/(L/c)$, $x^* = x/L$, $p^* = p/p_N$, $\rho^* = \rho/\rho_N$, $u^* = u/c$, and $m^* = u^*\rho^*$, which are introduced to generalize the problem, become

$$(p_{i+1}^{n+1*} + p_i^{n+1*} - p_{i+1}^{n*} - p_i^{n*}) + 2(m_{i+1}^{n+1*} - m_i^{n+1*})\Delta t^*/\Delta x^* = 0 \quad (4a)$$

$$(m_{i+1}^{n+1*} + m_i^{n+1*} - m_{i+1}^{n*} - m_i^{n*}) + 2(p_{i+1}^{n+1*} - p_i^{n+1*})\Delta t^*/ \Delta x^* + F^*(m_{i+1}^{n+1*} + m_i^{n+1*}) = 0 \quad (4b)$$

$$F^* = (fL/2D)\Delta t^* |m^*|/p^* \quad (4c)$$

The second power of m in the friction term of Equation 4b complicates the procedure of stability analysis. To avoid this complexity, the positive value F^* given by Equation 4c is introduced. In Equation 4b, the gravity term is neglected. By considering p and m expressed in the Fourier expansion form in space, each Fourier component is written as

$$p_i^{n*} = V_p^n e^{ji\theta} \quad (5a)$$

$$n_i^{n*} = V_m^n e^{ji}\theta \quad (5b)$$

where V_p^n and V_m^n are the amplitude functions at time level n of a particular component whose phase angle is θ. The phase angle is defined as $\theta = k\Delta x^*$ by using the wave number k, and j represents $\sqrt{-1}$. Consequently, substituting Equations 5a and 5b into Equations 4a and 4b gives the following equations:

$$\begin{bmatrix} V_p^{n+1} \\ V_p^n \end{bmatrix} = W \begin{bmatrix} V_p^n \\ V_p^{n-1} \end{bmatrix} \quad (6a)$$

$$W = \begin{bmatrix} a, & b \\ 1, & 0 \end{bmatrix} \quad (6b)$$

$$a = (2 + F^*)/(1 + F^* + K^*) \quad (6c)$$

$$b = -1/(1 + F^* + K^*) \quad (6d)$$

$$K^* = \{(2\Delta t^*/\Delta x^*) \sin\theta/(1 + \cos\theta)\}^2 \quad (6e)$$

In these expressions, W represents the amplification matrix. The stability criterion is that the complex eigenvalue λ of W must be less than or equal to one. The eigenvalue λ of Equation 6b becomes

$$\lambda = \{(1 + F^*/2) \pm \sqrt{(F^*/2)^2 - K^*}\}/(1 + F^* + K^*) \quad (7)$$

Since both F^* and K^* are positive values according to Equations 4c and 6e, the conditions of $|\lambda| \le 1$ is derived. This

Notation

A	Section area of pipe (m2)
c	Sound speed in isothermal condition (m/s)
D	Pipe diameter (m)
f	Darcy–Weisbach friction factor
ΔG	Change of mass in control volume V around a joint over the time step Δt (kg)
L	Pipe length (m)
M	Consumption at a joint (kg/s)
m	Mass flux (= $u\rho$) (kg/s.m2)
N	Maximum number of sections of a pipe or maximum number of joints in a network
p	Absolute pressure (Pa)
p_N	Atmospheric pressure (Pa)
R	Gas constant (m2/s2.K)
T	Absolute temperature (K)
Δt	Time step for computation (s)
u	Flow velocity (m/s)
V	Control volume around a joint (m3)
Δx	Length of a section for computation (m)

Greek symbols

ρ	Density (kg/m3)
ρ_N	Density at the normal conditions (kg/m3)

Subscripts

i	Node number of a pipe or pipe number connecting a joint
j	Section number of a pipe
N	Maximum section number of a pipe
t	Partial differentiation by time t
x	Partial differentiation by distance x

Superscript

n	Time level
*	Designation of dimensionless expression

Note: Flow rate m3/h is shown in the normal conditions of 273 K, 0.1 MPa.

means that the basic differential equations of the present method are unconditionally stable. The same procedure of the stability analysis can be applied to the Crank–Nicolson method used by Streeter and Wylie (1969), which indicated that the stability criterion becomes a function of Δt^* and θ and that a large Δt^* related with θ yields the unstable condition $|\lambda| > 1$.

2.4. Iterative calculation method at joints

A real pipeline network involves junctions, consumer stations, valve stations, and other boundaries. Such elements yield new boundary conditions responding to operation modes. To consider these features of nonpipe elements complicates the application of the Newton–Raphson method to the whole network system. For this reason, Guy (1967) proposed an iterative calculation method for the joints between pipes. The present study generalizes the concept of Guy's method in order to calculate the nodes with a consumer station and arbitrary number of branches.

In Figure 2, the following equation of state is applied to the control volume V designated by the dotted line around the joint k:

$$(p^{n+1} - p^n)V = c^2\Delta G \tag{8}$$

where P is the pressure at the joint and the superscript n represents the time level. The change of mass ΔG in the control volume V over the time step Δt is given by

$$\Delta G = \Delta t\left\{\sum_{i=1}^{n} K_i(m_i^{n+1} + m_i^n)A_i/2 - M\right\} \tag{9}$$

where the subscript i indicates the pipe number connected to the joint k. Mass flux m_i of pipe i is evaluated at the nearest node to the joint. K_i is defined as $+1$ if the flow goes into the joint and -1 if the flow comes out of the joint. M represents a consumption at the joint. Substituting Equation 9 into Equation 8, the following equation is obtained:

$$(p^{n+1} - p^n)V + c^2\Delta t\left\{\sum_{i=1}^{n} K_i(m_i^{n+1} + m_i^n)A_i/2 - M\right\} = 0 \tag{10}$$

p^{n+1}, which satisfies Equation 10 at all the pipe ends forming the joints, can be calculated in an iterative manner by using the bisection method (see Osiadacz 1984) or by Guy's method, which is used in the present study (see Guy 1967). Figure 3 shows the flow chart of the calculation.

3. Comparison with other methods using a simple line

3.1. Definition of the example

Let us suppose a simple straight line of 5 km in length and 500 mm internal diameter, holding a gas of molecular weight 18.0 at a pressure of 5 MPa. Now the outlet valve opens, and

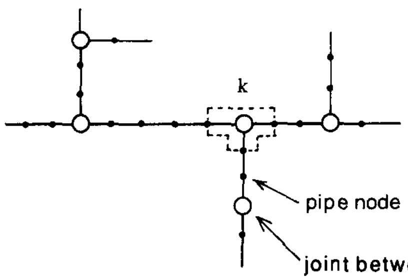


Figure 2 Definition sketch of pipe node and joint

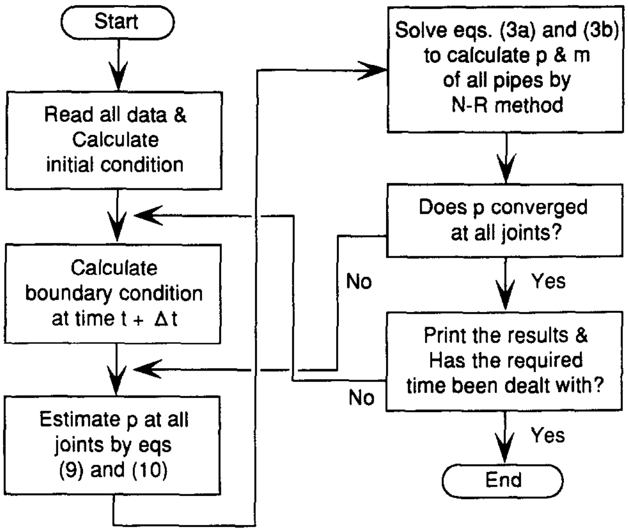


Figure 3 Flow chart of the present method

the outflow steps up from zero to 300,000 m^3/h while the inlet pressure is maintained at 5 MPa. After maintaining this condition for 20 minutes, the outlet valve closes. The friction factor f is assumed to be 0.008 for the calculation and the pipeline is not divided into sections in order to demonstrate the instability that often occurs due to an insufficient number of sections.

3.2. Comparison with the Crank–Nicolson method

The partially implicit algorithm of the Crank–Nicolson method in time, which does not always give a stable solution according to the Neumann stability analysis, is applied to the example in section 3.1. Figure 4a-1 and 4a-2 show the results of the Crank–Nicolson method used by Streeter and Wylie (1969). Figure 4b-1 and Figure 4b-2 show the results of the proposed method. As shown in Figure 4a-2, the Crank–Nicolson method gives an unstable solution in the case of a large time step. And even in the case of a small time step as shown in Figure 4a-1, the method gives unrealistic oscillation, as described in the following section.

3.3. Comparisons with MOC, the two-step Lax–Wendroff method, and Guy's implicit method

Programs based on two well-known explicit methods, i.e., MOC (Wylie and Streeter 1978) and the two-step Lax–Wendroff method (Roache 1985; Poloni 1987), are developed for comparison with the proposed method. Here, the two-step Lax–Wendroff method includes the second inertia term, and no technique is used to suppress numerical overshoots. These explicit methods give a correct answer when pipes are divided into sufficiently small sections. Guy's implicit method, which has not been previously investigated in comparison with other methods by any author, is also compared in the same manner as in the above discussion.

Since the calculation results of MOC using 8 and 25 sections agree well with each other, the eight sections are determined sufficient to simulate the transient for other methods. Guy's method uses nine sections, since the method requires an odd number of sections for the boundary condition of inlet pressure and outlet flow.

Figure 5a and 5b show the results of the two explicit methods. These results agree closely with each other in spite

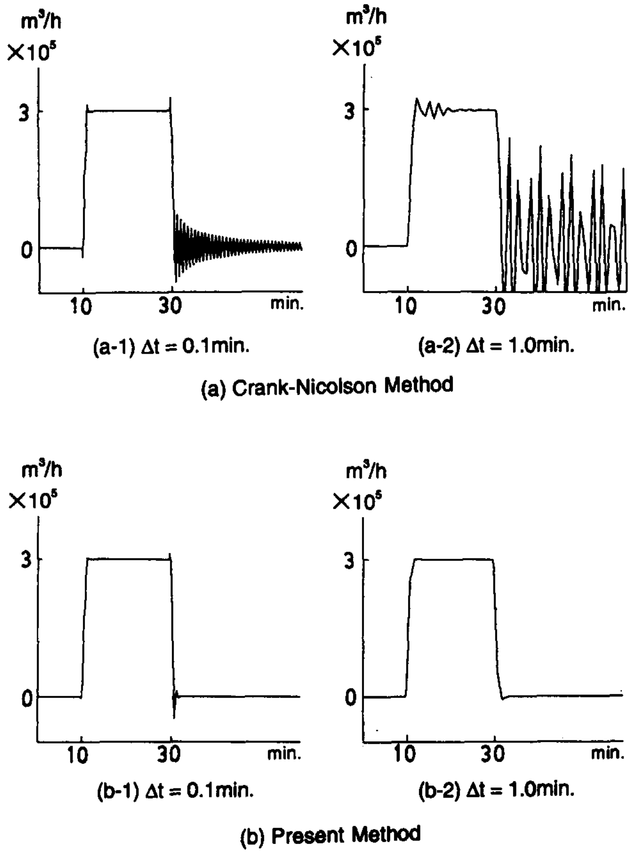


Figure 4 Comparison of the Crank–Nicolson method (a) with the present method (b)

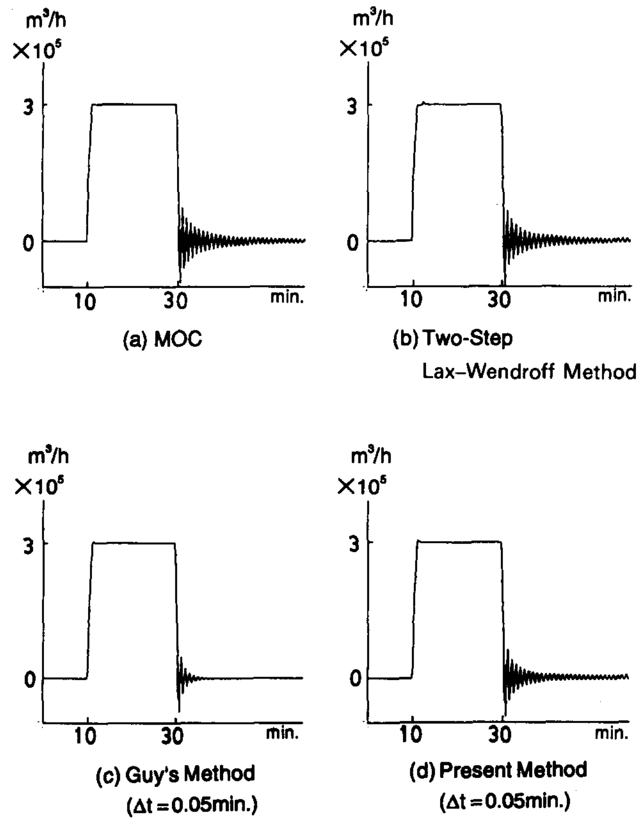


Figure 5 Comparison with other methods

Table 1 Ratio of computation time for the example of section 3

Present method ($\Delta t = 1$ min.)	1
Present method ($\Delta t = 0.05$ min.)	30
MOC	28
Two-step Lax–Wendroff method	29
Guy's method ($\Delta t = 0.05$ min.)	35

of neglecting the second inertia term in MOC. The calculated frequency of the oscillation after closure of the outlet valve is 0.0173 Hz. This coincides with a quarter wavelength oscillation of 0.0174 Hz given by $c/(4L)$, where the sound speed c is 348.5 m/s and the pipe length L is 5000 m.

Figures 5c and 5d show the results of two implicit methods. The oscillation is greatly damped in Guy's method. The same tendency can be seen in the proposed method when a small number of sections and a large time step are adopted, as shown in Figures 4b-1 and 4b-2. The frequencies of the oscillation are 0.0177 Hz in the proposed method and 0.0182 Hz in Guy's method. However, the frequency in Figure 4a-1 of the Crank–Nicolson method is 0.023 Hz.

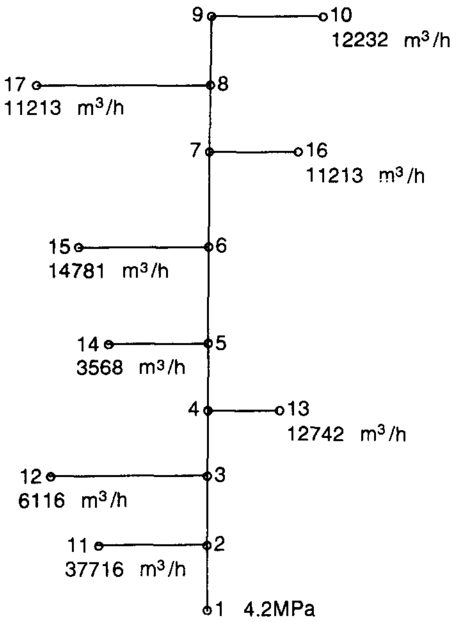
The above results indicate that the proposed method requires a small time step and a large number of sections if the problem to be solved involves rapid transients and the subject of interest concentrates on such phenomena. In other words, if these conditions apply, the method loses the advantage of fast computation, as shown in Table 1. However, in most cases of long-term computations of industrial gas pipelines, we are not concerned with such short-term rapid phenomena but rather with long-term transients. Consequently, small time steps and large numbers of sections are seldom required from a practical viewpoint.

4. An example of application to a network

Although the proposed method is already used in real large-city gas networks and has been proven to agree with the actual field measurements, the presentation of the results and operational conditions of such large complex networks is too tedious for our discussion. For this reason, Guy's (1967) sample calculation is chosen as an example of network calculation.

Figure 6 shows the network configuration and its dimensions. The network is assumed to start with steady conditions in which the inlet pressure at point 1 is 4.2 MPa and the outflows in the normal conditions at points 10 to 17 are written in the figure. All the outflows are then doubled over a period of half an hour, after which they remain at this increased value. The inflow at point 1 remains constant throughout the simulation. The fluid is methane gas, and the friction factor is set according to the Moody diagram. After 3.5 hours the pressure distribution is computed. For comparison, MOC is used again, where the shortest pipe is divided into four sections to obtain the accurate solution. A single section is adopted for the proposed method, and three sections for Guy's method, since Guy's method requires more than two sections for accuracy.

Figure 7 shows the pressure distribution along point 1 to 10 after 3.5 hours. The results of the two implicit methods agree with that of MOC. Even if the time step is increased from 1 minute to 30 minutes for the two implicit methods, almost the same results as those of MOC are obtained, and the difference in the calculated pressure at point 10 between the proposed method and MOC is 1.8 percent. Table 2 shows the ratio of computation time. It can be seen that the computation time is reduced by the proposed method.



	D (mm)	L (Km)		D (mm)	L (Km)
1-2	437	18.5	2-11	335	29.8
2-3	437	39.4	3-12	203	78.8
3-4	437	20.1	4-13	203	11.3
4-5	437	20.1	5-14	152	13.7
5-6	437	107.0	6-15	335	16.9
6-7	437	103.0	7-16	305	16.1
7-8	437	18.5	8-17	335	38.6
8-9	437	30.6			
9-10	305	12.9			

Figure 6 An example network for calculation

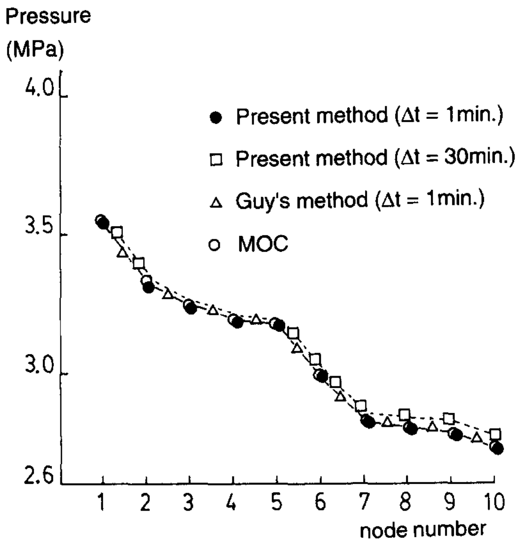


Figure 7 Calculated pressure distribution at 3.5 hours

5. Convergence at joints

Although the proposed method applied to a pipe is unconditionally stable, the iterative calculation at joints does not always assure the convergence. To obtain practical

Table 2 Ratio of computation time for the example of section 4

Present method ($\Delta t = 30$ min.)	1
Present method ($\Delta t = 1$ min.)	7.2
Guy's method ($\Delta t = 30$ min.)	4.3
MOC ($N = 4$)	103

knowledge on this feature, the following studies are performed again in the simple line described in section 3.1. The pipeline is divided into several pipes with joints. The convergence tolerance of 10 Pa is applied to all joints.

Figure 8 shows how the two parameters fL/D and Δt^* relate to the convergence. Here, fL/D is a dimensionless parameter relating pipeline size, and Δt^* is the dimensionless timestep. These quantities are defined in the dimensionless equations (Equations 4a and 4b). N denotes the number of joints. The maximum number of iterations is limited to 100. Range A, in Figure 8, refers to the region where the convergence is attained within the maximum number of iterations. In range B, solutions are obtained although the calculation is interrupted at the maximum number of iterations. In this range, the convergence tolerance is mitigated. In range C, the computation is terminated due to floating error. Although the accuracy of the simulation in range B is not investigated in this study, practically reasonable solutions are obtained in range B near to range A. These features imply that a large value of fL/D stabilizes the computation.

Figure 9 shows the relation between the time step Δt^* and the maximum number of iterations. A large time step can be used if the maximum number of iterations is large enough.

The above finding may be applied to the example of section 4, which has the dimensionless parameter fL/D of 13,200. It can be expected that this large fL/D makes the computation stable. Since the number of joints of this example is almost the same as that of Figure 9, the maximum time step of 5,460 seconds for convergence is estimated under the given limit of iterations of 500. The convergence is confirmed even with such large time steps. Therefore, the dimensionless relation in Figure 9 could be a fairly valid generalization.

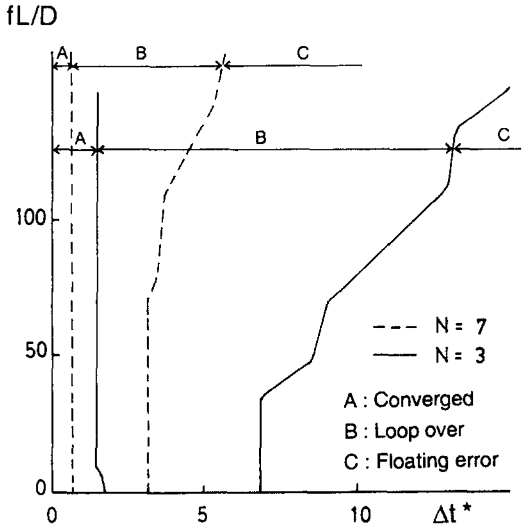


Figure 8 Influence of fL/D and dimensionless time step on convergence (the limited number of iteration = 100)

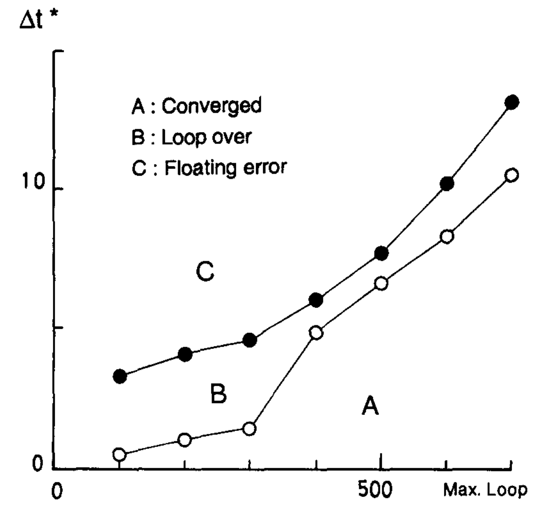


Figure 9 Influence of dimensionless time step and limited number of iteration on convergence ($N = 7$, $fL/D = 1$)

6. Conclusions

A fully implicit finite-difference method for the calculation of unsteady gas flow in pipeline networks is presented. Through the stability analysis and the comparison to other methods, the following conclusions are derived:
(1) The proposed method applied to a pipe is unconditionally stable. The convergence of the calculation at joints by using the iterative convergence method depends on the time-step, the number of iterations, and the pipeline size. However, it is shown that the convergence is predictable in most cases.
(2) The adoption of a small number of sections filters the local oscillatory responses in time and space. These responses can be calculated using a sufficient number of sections and a small time step, and agree with the results of traditional explicit methods. Since such local responses are less important in the computation of large complicated networks, the method practically gives good solutions using a large time step.
(3) Relative to Guy's implicit method, MOC, and the two-step Lax–Wendroff method, the proposed method can reduce the computation time.

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